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UNSTEADY RADIATIVE–CONVECTIVE HEAT TRANSFER IN A FLOW EMITTING-ABSORBING AND SCATTERING MEDIUM AROUND AN ABLATING PLATE

N. A. Rubtsov and V. A. Sinitsyn

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A conjugate problem of radiative–convective heat transfer in a turbulent high-temperature gas-disperse flow around a thermally thin ablating plate is considered. The plate experiences intense radiative heating by an external source, which is a blackbody. The temperature fields and the distributions of heat fluxes along the plate under unsteady conditions are calculated. The data gained make it possible to examine the effect of the Stark number and phase-transition heat in the plate material on the time evolution of the thermal state of the boundary-layer medium and the plate itself being heated by a high-temperature radiation source.

Key words: *radiation, turbulence, boundary layer, ablation, scattering.*

Radiative–convective heat transfer on a porous plate with blowing was considered previously [1–3]. The mass flux injected into the flow was independent of the plate temperature and was set *a priori*.

Mass supply into the boundary layer through the surface interrelated with heat transfer is studied in the present paper on the basis of the ablating-plate model.

We consider a conjugate problem of radiative–convective heat transfer in a turbulent flow of an emitting-absorbing and scattering gas-disperse medium around a thermally thin ablating plate. For simplicity, we assume that the plate-material vapor does not affect the optical and thermophysical properties of the medium. Next, we assume that the particles present in the flow have no effect on the thermophysical properties of the medium but determine its optical properties. In the course of the heat-transfer process, the particle size remains unchanged. The optical properties of the medium depend on temperature and radiation wavelength. The heat capacity is assumed to be constant; the viscosity and thermal conductivity are linear functions of temperature and the density varies reciprocally to temperature. Radiation transfer along the plate is ignored. The boundary-layer heating time is assumed to be much shorter that the plate heating time; for this reason, the boundary-layer heat transfer can be treated in a quasi-steady approximation. The initial plate temperature is T_{w0}; over the length $0 < x < x_0$, the temperature is maintained constant throughout the whole heating process. The lower surface and the trailing edge of the plate are thermally insulated. The source of radiation, which is a blackbody with a temperature T_s, is located outside the boundary layer. We consider radiation in a restricted spectral range Δ, where the medium absorbs and scatters radiation. The emitting surface of the source is parallel to the plate.

The thermal state of the plate is governed by an unsteady heat conduction equation, and boundary-layer heat transfer is described by a well-known set of equations including the continuity equation, the equation of motion, and the energy equation.

With the adopted assumptions, the dynamic problem reduces to the solution of the differential equation

$$((1+\mu_{\rm t})f'')' + \frac{1}{2}\,ff'' = \xi\Big(f'\,\frac{\partial f'}{\partial\xi} - f''\,\frac{\partial f}{\partial\xi}\Big) \tag{1}$$

with the boundary conditions

$$\eta = 0:\quad f = 0,\ f' = -f_w, \qquad \eta \to \infty:\quad f' \to 1.$$

Kutateladze Institute of Thermophysics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090.
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Here f is the dimensionless stream function, $f_w = V_w(\mathrm{Re}\,\xi)^{1/2}$, $V_w = \rho_w v_w/(\rho_\infty u_\infty)$ is the dimensionless surface mass flux defined below, the subscripts w and ∞ refer to the conditions at the plate and in the free flow,

$$\eta = \Big(\frac{\rho_\infty u_\infty}{\mu_\infty x}\Big)^{1/2} \int\limits_0^y \frac{\rho}{\rho_\infty}\,dy \qquad \text{and} \qquad \xi = x/L$$

are the transverse and streamwise dimensionless coordinates, x and y are the corresponding dimensional coordinates, u is the longitudinal velocity, ρ is the density, μ is the viscosity, L is the calculation length of the plate, and $\mathrm{Re} = \rho_\infty u_\infty L/\mu_\infty$ is the Reynolds number; the prime denotes differentiation with respect to η.

The thermal problem consists of equations and boundary conditions for heat transfer:

$$\frac{\partial}{\partial\eta}\Big(\Big(\frac{1}{\mathrm{Pr}} + \frac{\bar{\mu}_{\mathrm{t}}}{\mathrm{Pr}_{\mathrm{t}}}\Big)\frac{\partial\theta}{\partial\eta}\Big) + \frac{f}{2}\frac{\partial\theta}{\partial\eta} - \xi f'\frac{\partial\theta}{\partial\xi} - \frac{\mathrm{Sk}}{\mathrm{Re}\,\mathrm{Pr}}\xi\Psi = 0, \qquad \xi_0 < \xi < \xi_1, \quad 0 < \eta < \infty; \tag{2}$$

$$\xi = \xi_0\colon \quad \theta = \theta_0, \qquad \eta = 0\colon \quad \theta = \theta_w, \qquad \eta \to \infty\colon \quad \theta \to 1$$

in the boundary layer and

$$\frac{\partial\theta_w}{\partial\mathrm{Fo}} = \frac{\partial^2\theta_w}{\partial\xi^2} - æ\mathrm{Sk}Q_w, \qquad \xi_0 < \xi < \xi_1, \quad \mathrm{Fo} > 0; \tag{3}$$

$$\mathrm{Fo} = 0\colon \quad \theta_w = \theta_{w0}, \qquad \xi = \xi_0\colon \quad \theta_w = \theta_{w0}, \qquad \xi = \xi_1\colon \quad \frac{\partial\theta_w}{\partial\xi} = 0$$

in the plate. Here and below, $\bar{\mu}_{\mathrm{t}} = \mu_{\mathrm{t}}/\mu$ (μ_{t} is the turbulent viscosity), $\theta = T/T_\infty$ is the dimensionless temperature, $\theta_0(\eta)$ is the self-similar solution of the energy equation (2) without radiation, $æ = \lambda_\infty L/(\lambda_c H)$ is the conjugation parameter, H is the plate thickness, $\mathrm{Fo} = a_c t/L^2$, $\mathrm{Pr} = \mu_\infty/(\rho_\infty a_\infty)$, and $\mathrm{Sk} = 4\sigma T_\infty^3 L/\lambda_\infty$ are the Fourier number, Prandtl number, and Stark number, respectively, Pr_{t} is the turbulent Prandtl number, λ_c and λ_∞ are the thermal conductivities of the plate material and the medium in the free-stream flow, respectively, a_c and a_∞ are the thermal diffusivities of the plate material and of the free-stream flow, $\xi_0 = x_0/L$, $\xi_1 = x_1/L$, x_0 and x_1 are the boundaries of the calculation length of the plate, and σ is the Stefan–Boltzmann constant.

The dimensionless total heat-flux density on the plate Q_w in Eq. (3) is determined by the formula

$$Q_w = -\frac{1}{\mathrm{Sk}}\Big(\frac{\mathrm{Re}}{\xi}\Big)^{1/2}\frac{\partial\theta}{\partial\eta}\Big|_{\eta=0} + \Phi_w - \frac{\mathrm{Re}\,\mathrm{Pr}}{\mathrm{Sk}}V_w Q_L,$$

where $\Phi_w = E_w/(4\sigma T_\infty^4)$ (E_w is the integral total flux density of radiation on the plate) and $Q_L = q_L/(\rho_\infty c_p T_\infty)$ (q_L is the evaporation heat of the plate material).

The expression for the dimensionless divergence of the radiative flux density in Eq. (2) has the form

$$\Psi = \int\limits_\Delta \frac{\tau_{\lambda L}(E_{0\lambda} - E_{*\lambda})}{4\sigma T_\infty^4}\,d\lambda, \tag{4}$$

where $E_{0\lambda}(T) = 2\pi hc^2/[\lambda^5(\exp(hc/(k\lambda T)) - 1)]$ is the blackbody radiation flux density, $E_{*\lambda} = 2\pi\int\limits_{-1}^{1} I_\lambda(\tau_\lambda,\chi)\chi\,d\chi$ is the volume density of the incident radiation flux, I_λ is the intensity, χ is the cosine of the angle between the ordinate axis and the direction of radiation propagation, λ is the wavelength, c is the speed of light in vacuum, h and k are the Planck and Boltzmann constants, respectively, $\tau_{\lambda L} = k_\lambda L$ is the characteristic optical thickness, and k_λ is the attenuation factor of the medium; the subscript λ refers to spectral quantities. Integration over the wavelength in Eq. (4) is performed throughout the spectral range Δ, in which the medium absorbs and scatters radiation. The optical thickness in the boundary-layer cross section ξ is a function of wavelength and temperature given by the formula $\tau_\lambda = \Big(\frac{\xi}{\mathrm{Re}}\Big)^{1/2}\int\limits_0^\eta \frac{\tau_{\lambda L}}{\theta}\,d\eta$.

The radiative heat transfer in the system under study, which is a plane layer of an emitting-absorbing and scattering medium confined between the source and plate surfaces, is governed by the radiation-transfer equation. To solve this equation, the method of average fluxes is used [4]. The velocity field in the turbulent boundary layer is calculated by the double-layer Cebeci–Smith model [5].

Since the sought quantity (plate temperature) enters the boundary conditions for Eqs. (1) and (2), Eqs. (1) and (3) were solved together with the radiation-transfer equation by consecutive refinement of the plate temperature; Eq. (1) was integrated by the iteration-difference method [6].

The medium under study was a mixture of gaseous carbon dioxide, water vapor, and solid particles. The solid phase was coal and ash particles. With such a mixture, the steam boiler furnace atmosphere can be modeled to a certain extent.

Scattering in the gas phase being neglected, the attenuation factor of the model medium can be written as

$$k_\lambda = k_{\lambda p} + k_{\lambda g},$$

where $k_{\lambda p}$ is the attenuation factor of the particle cloud and $k_{\lambda g}$ is the absorption factor of the gas.

To make allowance for selective absorption of radiation in the gas phase, we use the narrow-band method based on the Goody statistical model [7]. The Goody model implies that the absorption lines are randomly distributed in the frequency spectrum, and their intensities are distributed according to some (most frequently, exponential) law. Within the framework of this method, the spectral absorption coefficient under moderate pressures can be represented as

$$k_{\lambda g} = P(\gamma_{\lambda \mathrm{CO_2}} C_{\mathrm{CO_2}} + \gamma_{\lambda \mathrm{H_2O}} C_{\mathrm{H_2O}}),$$

where P is the total gas pressure, C are the molar concentrations of the components in the mixture, and $\gamma_{\lambda \mathrm{H_2O}}$ and $\gamma_{\lambda \mathrm{CO_2}}$ are the average intensities of individual lines in the absorption band of water vapor and carbon dioxide.

The band parameter γ_λ is temperature-dependent. In the present study, we used the values of this parameter in the temperature range of 300–1500 K; these values were borrowed from [8–10]. To calculate radiation transfer, the bands of 7250, 5331, and 3755 cm^{-1} for H_2O and the bands of 667 and 3715 cm^{-1} for CO_2 were taken into account.

The parameters that describe the optical properties of the particles were borrowed from [11]. Treating the particle cloud as a polydisperse mixture with a gamma-distribution in terms of size, Kim and Lior [11] obtained approximate formulas for attenuation and scattering factors as functions of the diffraction parameter $x = \pi \bar{d}/\lambda$ ($\bar{d}$ is the mean particle diameter).

Ablation of a wetted surface is known as substance entrainment into the boundary-layer flow due to phase transitions (melting, evaporation), mechanical erosion, thermal destruction, etc. In the present work, we believe that evaporation is the governing process. Evaporation is assumed to be essentially nonequilibrium, the saturated-vapor pressure in the flow being much lower than the saturation pressure at any surface temperature. Such a situation is typical of high-velocity flows. Experimental studies of evaporation of many materials showed that the vapor mass flux as a function of plate temperature can be represented by the Langmuir–Knudsen law for evaporation under essentially nonequilibrium conditions [12]:

$$V_w = \frac{a_1}{\sqrt{\theta_w}} \exp\Big(-\frac{a_2}{\theta_w}\Big). \qquad (5)$$

Here, a_1 and a_2 are coefficients independent of plate temperature:

$$a_1 = \frac{a P_{\mathrm{sat}}(T_\infty)}{\rho_\infty u_\infty} \sqrt{\frac{M}{2\pi R T_\infty}} \exp\Big(\frac{q_L M}{R T_\infty}\Big), \qquad a_2 = \frac{q_L M}{R T_\infty}.$$

In the above formulas, a is the accommodation coefficient, P_{sat} is the saturated-vapor pressure, M is the molecular weight of the vapor, and R is the gas constant. The values of a_1 and a_2 for various materials range widely.

The results described below were obtained for a free-stream temperature $T_\infty = 1000$ K, source temperature $T_s = 1500$ K, dimensionless source temperature $\theta_s = T_s/T_\infty = 1.5$, $a_1 = 10^{-3}$, and $a_2 = 10^{-2}$. The computations were performed for the following values of the governing parameters: $\theta_{w0} = 0.3$, $\mathrm{Pr} = 0.7$, $\mathrm{Pr_t} = 0.9$, and $\mathrm{Re} = 10^6$. The emissivity of the plate surface was assumed to equal 0.99. It was also assumed that $C_{\mathrm{CO_2}} = 0$ and $C_{\mathrm{H_2O}} = 1$. The total gas pressure was $P = 1$ atm and the conjugation factor was $æ = 1$; the solid particles in the flow were coal particles available in a concentration of $2 \cdot 10^{-7}$ m^{-3}, and their mean diameter was 10^{-4} m.

The calculations showed that, for the indicated concentration of particles in the flow, the contribution of the gas phase into radiation transfer is insignificant. The latter is explained by discreteness of the gas-absorption spectrum and by a small boundary-layer thickness.

Figure 1 shows the distributions of the temperature θ_w along the plate under steady conditions for various values of the radiative–convective Stark criterion (Sk) and the parameter Q_L, which characterizes the intensity of heat absorption during the phase transition. Curves 3 in Fig. 1 for different values of Q_L are coincident. A higher

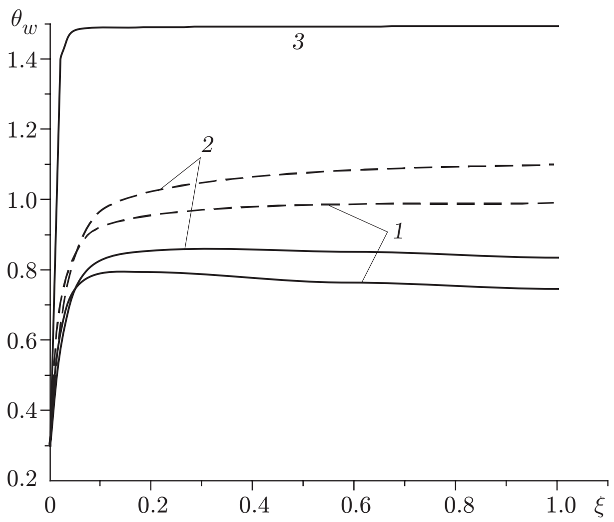

Fig. 1

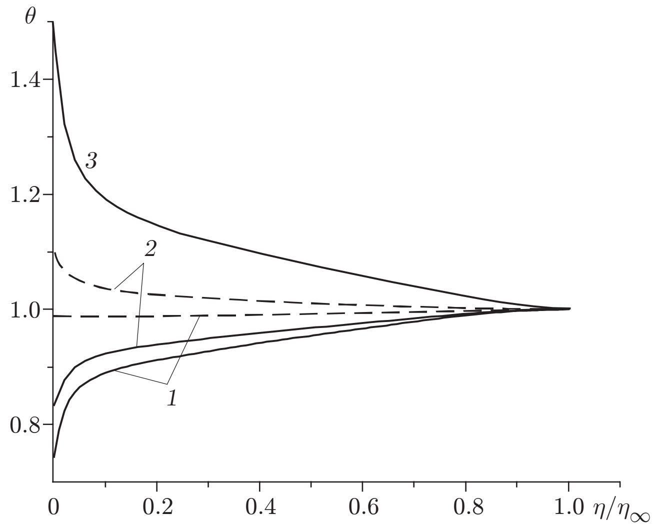

Fig. 2

Fig. 1. Temperature distribution along the plate under steady conditions for $\mathrm{Sk} = 10^2$ (1), 10^4 (2), and 10^6 (3): the dashed and solid curves refer to $Q_L = 0$ and 0.4, respectively.

Fig. 2. Temperature distribution in the boundary-layer cross section $\xi = \xi_1$ under steady conditions (notation the same as in Fig. 1).

level of temperatures corresponds to higher Stark numbers. The reason for this behavior of the temperature field is as follows. If the value of Sk is high, the major part in the heat exchange between the plate and the flow belongs to radiation. Under steady conditions, an intense flux of radiation from the plate is necessary to compensate for the radiation flux incident onto the plate from an external high-temperature field of radiation. This can only be achieved by plate heating. The calculation data show that an increase in the parameter Q_L results in plate-temperature reduction because the amount of heat absorbed during the phase transition increases. Here, stratification of the curves according to the parameter Q_L depends on the Stark number. At low values of Sk, the stratification is more pronounced, which can be explained by the fact that heat conduction appreciably affects heat transfer near the plate surface.

Figure 2 shows the distribution of the boundary-layer temperature θ in the last cross section over the coordinate ξ under steady conditions. The presence of an external source of radiation at high values of Sk leads to an appreciable overheating of the boundary layer in the vicinity of the wetted surface. At low values of Sk, the influence of the external source and heat absorption due to the plate-surface phase transition on temperature is manifested weakly. For $\mathrm{Sk} = 100$ and $Q_L = 0$, the external source of radiation gives rise to an isothermal (across the boundary layer) state. The same tendencies in the temperature curves are observed during variation of the Stark number and parameter Q_L. For intense radiative heat transfer (high values of Sk), the effect of heat absorption due to the phase transition on the boundary-layer temperature field under steady conditions is insignificant.

The performed computations make it possible to reveal the effect of heat absorption due to the phase transition on the distributions of heat fluxes along the plate. Figure 3 shows the total radiation flux density Φ_w versus Q_L at various times. Here, the dimensionless time (Fourier number) step is $\Delta\mathrm{Fo} = 2.5 \cdot 10^{-5}$. Curves 1 for different values of Q_L coincide. First of all, a decrease in the radiation flux with establishing steady conditions is worth noting. The reason is that the radiation emitted by the plate compensates for the incident radiation flux to a greater extent as the plate is heated. With increasing Q_L, the magnitude of Φ_w also increases. The latter is explained by the fact that the heat absorption due to the phase transition results in plate cooling and, hence, in reduced compensation of the incident radiation flux by the effective emission from the plate. Under steady conditions, this effect is manifested weaker.

Figure 4 shows the distributions of the total heat-flux density Q_w over the plate for different values of Q_L. The dimensionless time step is $\Delta\mathrm{Fo} = 5 \cdot 10^{-5}$. In the steady regime, the curves for different values of Q_L coincide. The ablation process is seen to exert the most pronounced influence on Q_w at the initial stage of heating, when the

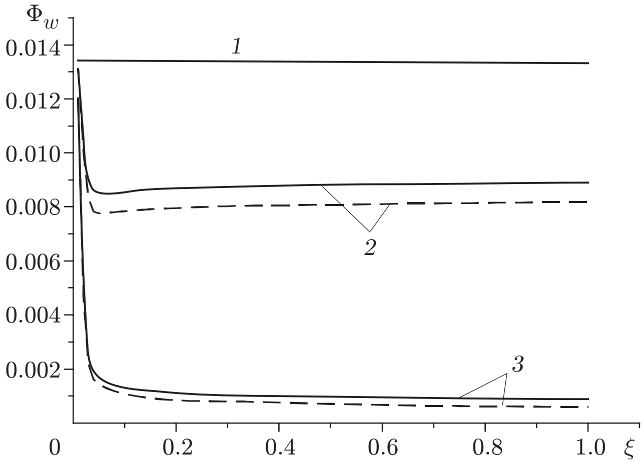

Fig. 3

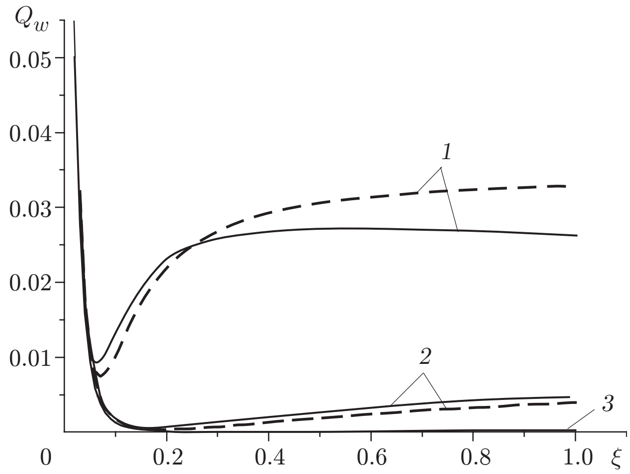

Fig. 4

Fig. 3. Effect of the parameter Q_L on the time evolution of the distribution of the total radiation flux density Φ_w over the plate: the dashed and solid curves refer to $Q_L = 0$ and 0.4; the calculation data were obtained for five time steps (1), for ten time steps (2), and in the steady regime (3).

Fig. 4. Effect of the parameter Q_L on the time evolution of the distribution of the total flux Q_w over the plate (notation the same as in Fig. 3).

plate-temperature distribution is strongly nonisothermal. Extreme regions are observed, where the total flux Q_w is minimal.

A preliminary analysis shows that the model proposed allows one to investigate the main features of boundary-layer heat- and mass-transfer processes in a flow of a high-temperature gas-disperse medium with an external source of radiation over a flat surface.

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